Memory-Event-Triggered H_∞ Filtering of Unmanned Surface Vehicles With Communication Delays

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***Abstract*—This brief studies the design of a memory-event-triggered H_∞ filter for unmanned surface vehicles (USVs) with communication delays. To save precious communication resources, a memory-event-triggered scheme (METS) is constructed by utilizing the mean of historical outputs. A novel united framework of the filtering error system under the METS and communication delays is established as a distributed delay system. By using the Bessel-Legendre (BL) inequality to handle the distributed delay, sufficient conditions are derived for designing H_∞ filters for USVs under METS. Finally, an illustrative example is simulated to show the advantage of the presented strategy.**

***Index Terms*—Memory-event-triggered scheme, H_∞ filtering, unmanned surface vehicles, distributed delay.**

I. Introduction

DUE TO the merits of high concealment, low cost and powerful operation, unmanned surface vehicles (USVs) have been applied in wide fields, such as environmental monitoring, scientific exploration and rescuing [1]–[3]. Note that the data communication of USVs is usually operated in a wireless network environment. With the introduction of communication networks, the network-induced problems including communication delays and limited bandwidth are inevitable. Reference [4] studies the fault detection filtering and control problem of networked USVs with communication delays. To save precious network resources, the event-triggered scheme (ETS) has gained many researchers' interest, which can be viewed as a substitute for the traditional time-triggered scheme. In [5], a switched time-delay model is presented for event-triggered unmanned aerial vehicles (UAVs) in cognitive radio networks. A discrete-time ETS is used in [6] for an underactuated USV with communication delays to improve bandwidth utilization.

On another research line, the filtering problem for practical systems is an interesting and fundamental issue in the control area. Some distributed event-triggered filtering/estimation methods, such as Kalman filtering, set-membership filtering and H_∞ filtering, are discussed in the survey [7]. Compared with Kalman filtering based on prior noise statistics, H_∞ filtering only needs the upper bounds of the input and measurement noises, which has attracted much attention from researchers and engineers [8]–[10]. For practical networked USVs under conventional ETSs dependent on instant system states or outputs in [5], [6], the disturbances or measurement noises induced by waves and winds may lead to tremendous data transmissions and waste limited bandwidth. To address this problem, [11] investigates the integral-based event-triggered control issue for stochastic linear systems. However, communication delays are not considered and the integral term caused by the average outputs is handled through an approximation method, which could result in approximation error. In addition, the threshold of ETS is related to the number of triggering events. For example, the number usually increases as the decrease of the triggering threshold parameter. Thus, it is practical and reasonable that the threshold should be adjusted adaptively to the system dynamics [12]. Unfortunately, the thresholds of ETSs in [5], [6] are constant scalars and cannot be adapted dynamically with the system dynamics.

Motivated by the aforementioned observations, this brief studies the H_∞ filtering issue of USVs with communication delays and the METS. The main contributions are as follows.

1) The mean of historical outputs over a fixed period is utilized as the input of the METS, which is helpful for decreasing unnecessary triggered events caused by disturbances and noises. Moreover, a dynamic triggering threshold parameter adapted with system dynamics is introduced to the METS, which is more practical than the existing static ETSs in [5], [6].

2) A novel united framework of the H_∞ filtering error system of a USV with communication delays and the METS is represented as a distributed delay system. Under this framework and a constructed LKF dependent on distributed delay, the BL inequality is used to obtain sufficient H_∞ filter design conditions under the METS.

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TABLE I
THE LIST OF VARIABLES IN (1)

$x_1(t)$	sway speed	$x_2(t)$	yaw speed
$x_3(t)$	heading angle	$x_4(t)$	roll speed
$x_5(t)$	roll angle	$\sigma(t)$	rudder angle
$\omega_1(t)$, $\omega_2(t)$	disturbances		

Notation: In this brief, $\mathbf{He}(P)$ means $P^T + P$. $\mathbf{Sy}(X, Y) \triangleq Y^T XY$. $\otimes$ denotes the Kronecker product.

II. PRELIMINARIES

The motion equations of a USV are given below [11]:

$$\begin{cases} \dot{x}_1(t) = -\frac{1}{\mathcal{T}_1}x_1(t) + \frac{\mathcal{K}_1}{\mathcal{T}_1}\sigma(t), \\ \dot{x}_2(t) = \frac{\mathcal{K}_2}{\mathcal{T}_2}x_1(t) - \frac{1}{\mathcal{T}_2}x_2(t) + \frac{\mathcal{K}_3}{\mathcal{T}_2}\sigma(t) + \frac{1}{\mathcal{T}_2}\omega_1(t) \\ \dot{x}_3(t) = x_2(t) \\ \dot{x}_4(t) = \varpi^2\mathcal{K}_4 x_1(t) - 2\lambda\varpi x_4(t) - \varpi^2 x_5(t) \\ +\varpi^2\mathcal{K}_5\sigma(t) + \varpi^2\omega_2(t) \\ \dot{x}_5(t) = x_4(t), \end{cases} \tag{1}$$

where the meanings of variables $x_i(t)$, $i = 1, \ldots, 5$, $\omega_1(t)$ and $\omega_2(t)$ are given in Table I. $\mathcal{K}_1$, $\mathcal{K}_2$, $\mathcal{K}_3$, $\mathcal{K}_4$, $\mathcal{K}_5$, $\mathcal{T}_1$, $\mathcal{T}_2$, λ and ϖ are known constant parameters.

By choosing $x(t) = [x_1(t), x_2(t), x_3(t), x_4(t), x_5(t)]^T \in \mathbb{R}^5$, $\omega(t) = [\omega_1(t), \omega_2(t)]^T \in \mathbb{R}^2$ and $u(t) = \sigma(t) \in \mathbb{R}^1$ for the state vector, disturbance vector and control input, respectively, the above motion equations are expressed as

$$\dot{x}(t) = A_0 x(t) + B_0 u(t) + B\omega(t), \tag{2}$$

where $B_0 = \begin{bmatrix} \frac{\mathcal{K}_1}{\mathcal{T}_1} & \frac{\mathcal{K}_3}{\mathcal{T}_2} & 0 & \varpi^2\mathcal{K}_5 & 0 \end{bmatrix}^T$,

$$A_0 = \begin{bmatrix} -\frac{1}{\mathcal{T}_1} & 0 & 0 & 0 & 0 \\ \frac{\mathcal{K}_2}{\mathcal{T}_2} & -\frac{1}{\mathcal{T}_2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \varpi^2\mathcal{K}_4 & 0 & 0 & -2\lambda\varpi & -\varpi^2 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ \frac{1}{\mathcal{T}_2} & 0 \\ 0 & 0 \\ 0 & \varpi^2 \\ 0 & 0 \end{bmatrix}.$$

In this brief, it is assumed that the USV has been stabilized by a state feedback controller, $u(t) = Kx(t)$, where the controller gain K is chosen such that $A_0 + B_0 K$ is Hurwitz. Then, the stabilized USV system is given as

$$\begin{cases} \dot{x}(t) = Ax(t) + B\omega(t) \\ y(t) = Cx(t) \\ z_s(t) = Dx(t), \end{cases} \tag{3}$$

where $A \triangleq A_0 + B_0 K$ is the stabilized system matrix, $y(t) \in \mathbb{R}^r$ is the measurement output, $z_s(t) \in \mathbb{R}^m$ is the performance output and $\omega(t)$ belongs to $\mathscr{L}_2[0, \infty)$.

In contrast to the conventional ETS based on instant measurements, the mean of past measured outputs over a given interval τ, $\bar{y}(s) \triangleq \frac{1}{\tau}\int_{s-\tau}^{s} y(v)dv$, is utilized as the input of the following METS:

$$s_{k+1} = \min_s\left\{s \geq s_k | e^T(s)\Phi e(s) \geq \delta(s)\mathbf{Sy}(\Phi, \bar{y}(s)) + \varsigma\right\}, \tag{4}$$

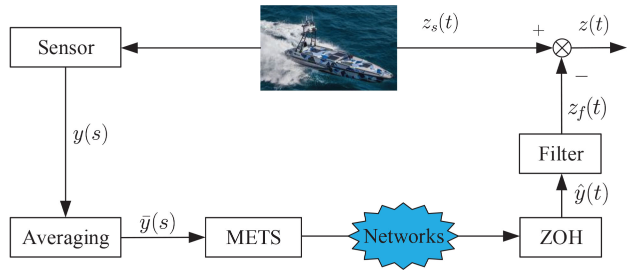

Fig. 1. The filtering framework for USV under the METS.

where s_k and s_{k+1} denote the present and next triggering instant, respectively, $\Phi > 0$ is the weighting matrix, $\varsigma > 0$ is a constant scalar, $e(s) = \bar{y}(s) - \bar{y}(s_k)$ and $\delta(s) \in [\delta_m, \ \delta_M]$ ($0 < \delta_m < \delta_M < 1$) is a dynamic triggering threshold parameter governed by the following adapting law:

$$\delta(s) = \delta_M - (\delta_M - \delta_m)e^{-\|\bar{y}(s)\|}. \tag{5}$$

The filtering framework for a networked USV with the METS is shown in Fig. 1.

Remark 1: In practical systems, the measurement outputs could contain some random fluctuations caused by disturbances or noises. Note that the conventional ETSs dependent on instant measurement output $y(t)$ may be sensitive to such fluctuations and generate many redundant data. To reduce the data communication induced by random fluctuations, the mean of historical outputs is used as the input of the triggering rule, which also inspires the name of METS.

Remark 2: If $\tau \to 0$, it is seen that $\lim_{\tau\to 0}\frac{1}{\tau}\int_{s-\tau}^{s} y(v)dv = y(s)$. Then the proposed METS becomes

$$s_{k+1} = \min_s\left\{s \geq s_k | \hat{\epsilon}^T(s)\Phi\hat{\epsilon}(s) > \delta\mathbf{Sy}(\Phi, y(s)) + \varsigma\right\}, \tag{6}$$

which is the conventional ETS in [13] with $\hat{\epsilon}(s) = y(s) - y(s_k)$ and $\delta = \delta_m = \delta_M$. Compared with [13], based on instant outputs, the METS using the average outputs has the potential ability to avoid redundant triggered signals induced by disturbances and noises. Moreover, with the introduction of the positive term ς, Zeno behavior (infinite triggered events in finite time) can be excluded naturally.

Remark 3: The adapting law of the dynamic triggering threshold parameter relies on the mean of the measured output $\bar{y}(s)$. When $\bar{y}(s)$ approaches the equilibrium point, fewer communication signals are needed to maintain the system performance, which requires a larger $\delta(s)$. When $\bar{y}(s)$ is away from the equilibrium point, one has to decrease $\delta(s)$ to generate more signals to improve the system performance.

By considering the network transmission delay and zero-order hold (ZOH), the input of filter $\hat{y}(t)$ ($t \in \Gamma \triangleq [t_k, \ t_{k+1})$) is expressed as

$$\hat{y}(t) = \bar{y}(s_k), \tag{7}$$

where $t_k = s_k + \eta$, $t_{k+1} = s_{k+1} + \eta$, η is the constant communication delay. Then, (7) is written as

$$\hat{y}(t) = \frac{1}{\tau}\int_{t-d}^{t-\eta} y(v)dv - \epsilon(t), \tag{8}$$

where $d = \eta + \tau$, $\epsilon(t) \triangleq \frac{1}{\tau}\int_{t-d}^{t-\eta} y(v)dv - \bar{y}(s_k)$.

To estimate the performance output $z_s(t)$, a memory-event-triggered filter is given as

$$\begin{cases} \dot{x}_f(t) = A_f x_f(t) + B_f \hat{y}(t) \\ z_f(t) = D_f x_f(t), \end{cases} \tag{9}$$

where $x_f(t) \in \mathbb{R}^5$ is the filter state, $z_f(t) \in \mathbb{R}^m$ is the filter output and A_f, B_f and D_f need to be determined.

By substituting (8) into (9), we have

$$\begin{cases} \dot{\xi}(t) = \mathcal{A}\xi(t) + \mathcal{B}\omega(t) + \mathcal{B}_1\epsilon(t) + \dfrac{\mathcal{B}_2}{\tau}\int_{t-d}^{t-\eta} x(v)dv \\ z(t) = \mathcal{D}\xi(t), \end{cases} \tag{10}$$

where $\xi(t) = [\, x^T(t) \quad x_f^T(t)\,]^T$, $z(t) = z_s(t) - z_f(t)$ and

$$\mathcal{A} = \begin{bmatrix} A & 0 \\ 0 & A_f \end{bmatrix}, \mathcal{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \mathcal{B}_1 = \begin{bmatrix} 0 \\ -B_f \end{bmatrix}, \mathcal{B}_2 = \begin{bmatrix} 0 \\ B_f C \end{bmatrix}, \mathcal{D}^T = \begin{bmatrix} D^T \\ -D_f^T \end{bmatrix}.$$

The aim of this brief is to design a memory-event-triggered H_∞ filter (9) satisfying.

a) Under $\omega(t) = 0$, the filtering error system (10) is uniformly ultimately bounded (UUB);

b) Under $\omega(t) \neq 0$ and $\xi(0) = 0$, $\int_0^\infty z^T(t)z(t)dt < \gamma^2 \int_0^\infty \omega^T(t)\omega(t)dt$ is satisfied with a given H_∞ index, $\gamma > 0$.

Before ending this section, the following definition and lemma are provided to derive the main results.

Definition 1 [14]: The Legendre polynomials defined over the interval $[a,\ b]$ are given as $L_i(v) \triangleq (-1)^i \sum_{j=0}^{i} \mathcal{P}_j^i \left(\dfrac{v-a}{b-a}\right)^j$ with $\mathcal{P}_j^i = (-1)^j \binom{i}{j}\binom{i+j}{j}$.

Three properties of Legendre polynomials are presented as

1) Orthogonality [14]: $\forall n \in \mathbb{N}$, $\int_a^b \mathcal{L}_n^T(v)\mathcal{L}_n(v)dv = \frac{1}{b-a}W_n$, where $\mathcal{L}_n(v) \triangleq [\, L_0(v) \quad \cdots \quad L_i(v) \quad \cdots \quad L_n(v)\,]^T$ and $W_n = diag\{1, 3, \ldots, 2n+1\}$;

2) Bound [14]: $\forall i \in \mathbb{N}$, $L_i(b) = 1$, $L_i(a) = (-1)^i$;

3) Differentiation [14]: For $i = 0$, $\dot{L}_i(v) = 0$; and for $i \geq 1$, $\dot{L}_i(v) = \sum_{k=0}^{i-1} \frac{(2i+1)}{b-a}(1-(-1)^{i+k})$.

Lemma 1 [14]: For a given function $x(v) \in \mathbb{R}^q$, $v \in [a,\ b]$, and a matrix $\mathcal{R} \in \mathbb{R}^{q\times q} > 0$, the following inequality

$$\int_a^b \mathbf{Sy}(\mathcal{R}, x(v))dv \geq \frac{1}{b-a}\mathbf{Sy}(W_n \otimes \mathcal{R}, \Omega_n) \tag{11}$$

holds for $\Omega_n = \int_a^b \mathbb{L}_n(v)x(v)dv$ and $\mathbb{L}_n(v) \triangleq \mathcal{L}_n(v) \otimes I_q$.

III. Main Results

Before giving the design of H_∞ filters of system (10), the following abbreviations are defined:

$$\mathscr{L}_n^1(t) = \int_{-\eta}^0 \mathbb{L}_n^1(v)x(t+v)dv,\ \mathscr{L}_n^2(t) = \int_{-d}^{-\eta} \mathbb{L}_n^2(v)x(t+v)dv,$$

$$\mathbb{I}_a = \begin{cases} \left[0_{5,5a}\ I_5\ 0_{5,5(8+2n-a)+r+2+m}\right], & a = 1,\ldots,8+2n \\ \left[0_{r,5(8+2n)} \quad I_r \quad 0_{r,2+m}\right], & a = 9+2n \\ \left[0_{2,5(8+2n)+r} \quad I_2 \quad 0_{2,m}\right], & a = 10+2n \\ \left[0_{m,5(8+2n)+r+2} \quad I_m\right], & a = 11+2n. \end{cases}$$

Theorem 1: For given scalars τ, δ_M, η and ς, the filtering error system (10) under the METS (4) is UUB and satisfies the H_∞ index γ, if there exist matrices $Q_i > 0$, $R_i > 0$ $(i = 1, 2)$, $\Phi > 0$, P, N_{11}, N_{12}, N_{13}, N_{21}, N_{23}, $\mathscr{A}_f$, $\mathscr{B}_f$, D_f, such that

$$\mathscr{P} > 0, \tag{12}$$

$$\Upsilon + \mathbf{He}(\Xi) < 0, \tag{13}$$

where $\mathscr{Q}_i = W_n \otimes Q_i$, $\mathscr{R}_i = W_n \otimes R_i$, $i = 1, 2$,

$$\mathscr{P} = P + diag\{0_{10}, \frac{1}{\eta}\mathscr{Q}_1, \frac{1}{\tau}\mathscr{Q}_2\}, \quad \mathscr{E} = \frac{C\mathcal{I}}{\tau}\mathbb{I}_n^2,$$

$$\begin{aligned} \Upsilon = {} & \mathbf{He}(\mathscr{M}^T P \mathscr{H}) + \delta_M \mathbf{Sy}(\Phi, \mathscr{E}) + \mathbf{Sy}(Q_1 + \eta R_1 \\ & + \varsigma^{\frac{1}{3}} I_5, \mathbb{I}_3) + \mathbf{Sy}(\varsigma^{\frac{1}{3}} I_5, \mathbb{I}_4) + \mathbf{Sy}(Q_2 - Q_1 - dR_2, \mathbb{I}_5) \\ & - \mathbf{Sy}(Q_2, \mathbb{I}_6) - \mathbf{Sy}(\Phi, \mathbb{I}_{9+2n}) - \gamma^2 \mathbf{Sy}(I_2, \mathbb{I}_{10+2n}) \\ & - \mathbf{Sy}(I_m, \mathbb{I}_{11+2n}) + \mathbf{He}\big(\mathbb{I}_{11+2n}^T(D\mathbb{I}_3 - D_f\mathbb{I}_4)\big) \\ & - \frac{1}{\eta}\mathbf{Sy}\Big(\mathscr{R}_1, \mathbb{I}_n^1\Big) - \frac{1}{\tau}\mathbf{Sy}\Big(\mathscr{R}_2, \mathbb{I}_n^2\Big), \end{aligned}$$

$$\mathbb{I}_n^1 = \begin{bmatrix} \mathbb{I}_7 \\ \vdots \\ \mathbb{I}_{7+n} \end{bmatrix},\quad \mathbb{I}_n^2 = \begin{bmatrix} \mathbb{I}_{8+n} \\ \vdots \\ \mathbb{I}_{8+2n} \end{bmatrix},\quad \mathscr{M} = \begin{bmatrix} \mathbb{I}_3 \\ \mathbb{I}_4 \\ \mathbb{I}_n^1 \\ \mathbb{I}_n^2 \end{bmatrix},$$

$$\mathscr{H} = \begin{bmatrix} \mathbb{I}_1 \\ \mathbb{I}_2 \\ \mathbf{1}\mathbb{I}_3 - \hat{\mathbf{1}}\mathbb{I}_5 - \frac{\Lambda_n}{\eta_M}\mathbb{I}_n^1 \\ \mathbf{1}\mathbb{I}_5 - \hat{\mathbf{1}}\mathbb{I}_6 - \frac{\Lambda_n}{\tau}\mathbb{I}_n^2 \end{bmatrix},\ \mathbf{1} = \begin{bmatrix} I_5 \\ \vdots \\ I_5 \end{bmatrix}_{5(n+1),5},$$

$$\hat{\mathbf{1}} = \begin{bmatrix} I_5 \\ \vdots \\ (-1)^n I_5 \end{bmatrix}_{5(n+1),5},\ \Lambda_n = \begin{bmatrix} \lambda_0^0 I & \cdots & \lambda_0^n I \\ \vdots & \lambda_g^i & \vdots \\ \lambda_n^0 I & \cdots & \lambda_n^n I \end{bmatrix},$$

$$\lambda_g^i = \begin{cases} -(2i+1)(1-(-1)^{g+i}), & i \leq g, \\ 0, & i > g, \end{cases}$$

$$\begin{aligned} \Xi = {} & \mathbb{I}_1^T\Big(-N_{11}\mathbb{I}_1 - N_{12}\mathbb{I}_2 + N_{11}A\mathbb{I}_3 + \mathscr{A}_f\mathbb{I}_4 \\ & \quad + \frac{\mathscr{B}_f C\mathcal{I}}{\tau}\mathbb{I}_n^2 - \mathscr{B}_f\mathbb{I}_{9+2n} + N_{11}B\mathbb{I}_{10+2n}\Big) \\ & + \mathbb{I}_2^T\Big(-N_{13}\mathbb{I}_1 - N_{12}\mathbb{I}_2 + N_{13}A\mathbb{I}_3 + \mathscr{A}_f\mathbb{I}_4 \\ & \quad + \frac{\mathscr{B}_f C\mathcal{I}}{\tau}\mathbb{I}_n^2 - \mathscr{B}_f\mathbb{I}_{9+2n} + N_{11}B\mathbb{I}_{10+2n}\Big) \\ & + \mathbb{I}_3^T\Big(-N_{21}\mathbb{I}_1 - N_{12}\mathbb{I}_2 + N_{21}A\mathbb{I}_3 + \mathscr{A}_f\mathbb{I}_4 \\ & \quad + \frac{\mathscr{B}_f C\mathcal{I}}{\tau}\mathbb{I}_n^2 - \mathscr{B}_f\mathbb{I}_{9+2n} + N_{11}B\mathbb{I}_{10+2n}\Big) \\ & + \mathbb{I}_4^T\Big(-N_{23}\mathbb{I}_1 - N_{12}\mathbb{I}_2 + N_{23}A\mathbb{I}_3 + \mathscr{A}_f\mathbb{I}_4 \\ & \quad + \frac{\mathscr{B}_f C\mathcal{I}}{\tau}\mathbb{I}_n^2 - \mathscr{B}_f\mathbb{I}_{9+2n} + N_{11}B\mathbb{I}_{10+2n}\Big). \end{aligned}$$

Furthermore, the filter gains are solved via

$$A_f = N_{12}^{-1}\mathscr{A}_f, \quad B_f = N_{12}^{-1}\mathscr{B}_f, \quad D_f = D_f.$$

Proof: Define $\zeta(t) = \left[\xi^T(t) \quad \mathscr{L}_n^{1T}(t) \quad \mathscr{L}_n^{2T}(t)\right]^T$. The LKF is chosen as

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{14}$$

where $V_1(t) = \mathbf{Sy}(P, \zeta(t))$,

$$V_2(t) = \int_{-\eta}^0 \mathbf{Sy}(Q_1 + (v+\eta)R_1, x(t+v))dv,$$

$$V_3(t) = \int_{-d}^{-\eta} \mathbf{Sy}(Q_2 + (v+d)R_2, x(t+v))dv.$$

The proof of condition (12) is the same as the process in [14], which is omitted here.

Next, the time derivative of $V(t)$ is calculated as

$$\dot{V}_1(t) = 2\zeta^T(t)P\dot{\zeta}(t), \tag{15}$$

$$\dot{V}_2(t) = \mathbf{Sy}(Q_1 + \eta R_1, x(t)) - \mathbf{Sy}(Q_1, x(t-\eta)) - \int_{-\eta}^{0} \mathbf{Sy}(R_1, x(t+v))dv, \tag{16}$$

$$\dot{V}_3(t) = \mathbf{Sy}(Q_2 + dR_2, x(t-\eta)) - \mathbf{Sy}(Q_2, x(t-d)) - \int_{-d}^{-\eta} \mathbf{Sy}(R_2, x(t+v))dv. \tag{17}$$

Based on the properties of $L_i(v)$, we have

$$\dot{\mathscr{L}}_n^1(t) = \mathbf{1}x(t) - \hat{\mathbf{1}}x(t-\eta) - \frac{\Lambda_n}{\eta}\mathscr{L}_n^1(t), \tag{18}$$

$$\dot{\mathscr{L}}_n^2(t) = \mathbf{1}x(t-\eta) - \hat{\mathbf{1}}x(t-d) - \frac{\Lambda_n}{\tau}\mathscr{L}_n^2(t). \tag{19}$$

Applying Lemma 1 results in

$$-\int_{-\eta}^{0} \mathbf{Sy}(R_1, x(t+v))dv \le -\frac{1}{\eta}\mathbf{Sy}\Big(\mathscr{R}_1, \mathscr{L}_n^1(t)\Big), \tag{20}$$

$$-\int_{-d}^{-\eta} \mathbf{Sy}(R_2, x(t+v))dv \le -\frac{1}{\tau}\mathbf{Sy}\Big(\mathscr{R}_2, \mathscr{L}_n^2(t)\Big). \tag{21}$$

By denoting $\chi(t) = [\dot{\xi}^T(t), \xi^T(t), x^T(t-\eta), x^T(t-d), \mathscr{L}_n^{1^T}(t), \mathscr{L}_n^{2^T}(t), \epsilon^T(t), \omega^T(t), z^T(t)]^T$, it gives

$$\zeta(t) = \mathscr{M}\chi(t), \ \dot{\zeta}(t) = \mathscr{H}\chi(t). \tag{22}$$

To guarantee the H_∞ stability of system (10), it requires

$$\mathscr{J}(t) \triangleq \dot{V}(t) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) < 0. \tag{23}$$

From (4) and (8), we have

$$\epsilon^T(t)\Phi\epsilon(t) < \delta_M\mathbf{Sy}\left(\Phi, \frac{C}{\tau}\int_{-d}^{-\eta} x(t+v)dv\right) + \varsigma, \tag{24}$$

which can be reformed to

$$\delta_M\mathbf{Sy}(\Phi, \mathscr{E}\chi(t)) + \varsigma - \mathbf{Sy}(\Phi, \mathbb{I}_{9+2n}\chi(t)) > 0. \tag{25}$$

Then (23) is ensured if the following inequality is satisfied:

$$\begin{aligned}&\mathscr{J}(t) + \delta_M\mathbf{Sy}(\Phi, \mathscr{E}\chi(t)) - \mathbf{Sy}(\Phi, \mathbb{I}_{9+2n}\chi(t)) \\ &\quad + \varsigma - \varsigma^{\frac{1}{3}}\xi^T(t)\xi(t) + \varsigma^{\frac{1}{3}}\xi^T(t)\xi(t) \\ &\le \chi^T(t)\Upsilon\chi(t) + \varsigma - \varsigma^{\frac{1}{3}}\xi^T(t)\xi(t) < 0.\end{aligned} \tag{26}$$

From the definition of $\chi(t)$, system (10) is expressed as

$$\mathscr{G}\chi(t) = 0, \tag{27}$$

where $\mathscr{G} = -\begin{bmatrix}\mathbb{I}_1\\ \mathbb{I}_2\end{bmatrix} + \mathcal{A}\begin{bmatrix}\mathbb{I}_3\\ \mathbb{I}_4\end{bmatrix} + \frac{\mathcal{B}_2\mathcal{I}}{\tau}\mathbb{I}_n^2 + \mathcal{B}_1\mathbb{I}_{9+2n} + \mathcal{B}\mathbb{I}_{10+2n}$.

Constructing $\mathbf{N}_1 = \begin{bmatrix}N_{11} & N_{12}\\ N_{13} & N_{12}\end{bmatrix}$, $\mathbf{N}_2 = \begin{bmatrix}N_{21} & N_{12}\\ N_{23} & N_{12}\end{bmatrix}$, $\mathscr{N} = \begin{bmatrix}\mathbb{I}_1\\ \mathbb{I}_2\end{bmatrix}^T\mathbf{N}_1 + \begin{bmatrix}\mathbb{I}_3\\ \mathbb{I}_4\end{bmatrix}^T\mathbf{N}_2$ such that $\mathscr{N}\mathscr{G}\chi(t) = 0$ yields

$$\chi^T(t)(\Upsilon + \mathbf{He}(\mathscr{N}\mathscr{G}))\chi(t) + \varsigma - \varsigma^{\frac{1}{3}}\xi^T(t)\xi(t) < 0. \tag{28}$$

By defining $\mathscr{A}_f = N_{12}A_f$, $\mathscr{B}_f = N_{12}B_f$ and $D_f = D_f$ and substituting them into (28), we have $\mathscr{N}\mathscr{G} = \Xi$.

According to the [13, Definition 1], if $\|\xi(t)\| \le \varsigma^{\frac{1}{3}}$, the system (10) is said to be UUB.

Moreover, when $\|\xi(t)\| \ge \varsigma^{\frac{1}{3}}$, one can obtain $\varsigma - \varsigma^{\frac{1}{3}}\xi^T(t)\xi(t) \le 0$.

From $\Upsilon + \mathbf{He}(\Xi) < 0$ in (13) and $\mathscr{N}\mathscr{G} = \Xi$, the condition (28) is ensured, which guarantees $\mathscr{J}(t) < 0$.

For $\omega(t) = 0$, one has $\dot{V}(t) < -z^T(t)z(t) < 0$, which means the Lyapunov function $V(t)$ is monotonically decreasing. Then $\|\xi(t)\|$ will converge to less than $\varsigma^{\frac{1}{3}}$, which ensures system (10) to be UUB.

For $\omega(t) \ne 0$ and zero initial condition, by integrating $\mathscr{J}(t)$ over $[0, \infty)$, we have $\int_0^\infty z^T(t)z(t)dt < \gamma^2\int_0^\infty \omega^T(t)\omega(t)dt$. Then the proof is completed. ■

Remark 4: With the proposed METS, a novel closed-loop H_∞ filtering error system is modeled as a distributed delay system. The distributed delay can be approximated by some discrete delays via Simpson's rule [15], which will result in approximation error and design conservativeness. Therefore, how to deal with the distributed delay without introducing any approximation error is a challenge and difficult issue. In order to conquer this difficulty, the BL integral inequality (11) in Lemma 1 rather than Simpson's rule is utilized. Then, the distributed delay in (10) can be handled directly, which prevents the approximation error. Moreover, the BL inequality reduces to Jensen inequality and Wirtinger-based inequality with the degree of Legendre polynomials $n = 0$ and $n = 1$, respectively. As the increase of n, less conservative results may be obtained by BL integral inequality than existing Jensen inequality and Wirtinger-based inequality.

IV. Example

To show the effectiveness of the developed approach, the same parameters as [11] are chosen:

$$\mathcal{T}_v = 0.5263, \mathcal{T}_r = 0.4211, \mathcal{K}_{dr} = -0.0103, \mathcal{K}_{dp} = -0.0202,$$
$$\mathcal{K}_{dv} = 0.038, \mathcal{K}_{vp} = 0.798, \mathcal{K}_{vr} = -0.46, \omega_n = 1.63, \lambda = 2.084.$$

The controller $K = [1.1118, 3.465, 6.6327, -1.5691, 3.1662]$ is selected to stabilize the system in advance. Meanwhile, the system output matrix and performance output matrix are selected as $C = [1, 0.8, 1, -1, 0.6]$, $D = \begin{bmatrix}2 & 0 & 1 & 0 & 0\\ 0 & 1 & 0 & 2 & 0\end{bmatrix}$. The communication delay and the sampling time are considered as $\eta = 0.1s$ and $0.05s$, respectively.

For $\delta_m = 0.01$, $\delta_M = 0.2$, $\varsigma = 10^{-6}$ and the same H_∞ performance $\gamma = 1$, the filter gains for $\tau = 0.25$ and $\tau = 0.5$ can be solved by Theorem 1, respectively. In the simulation, we consider the disturbance as $\omega(t) = [sin(0.5\pi t); sin(0.5\pi t)]$ for $t \in [0, 6s]$ (otherwise, $\omega(t) = 0$). We also consider that the system output $y(t)$ contains the stochastic measurement noise, which is a standard Gaussian distributed random signal subject to $\mathcal{N}(0, 0.05)$. For $\tau = 0.25$ and $\tau = 0.5$, the curves of filtering error $e(t)$, the triggering instants and the triggering parameter threshold are depicted in Fig. 2 and Fig. 3, respectively. Them show that the designed filters can ensure the filtering error to be asymptotically stable.

The numbers of signal transmissions under different communication schemes are derived in Table II, in which $\mathcal{N}_1$

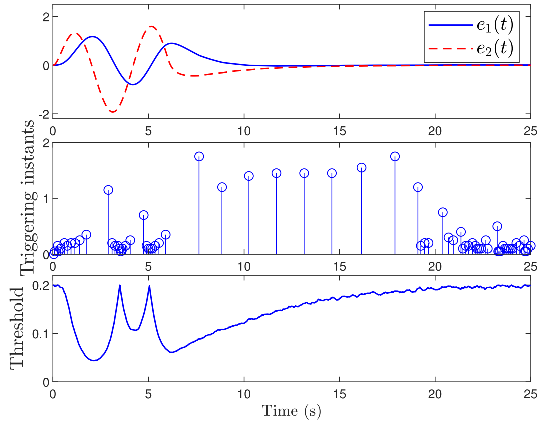

Fig. 2. Filtering error $e(t)$, triggering instants and triggering threshold parameter with $\tau = 0.25$.

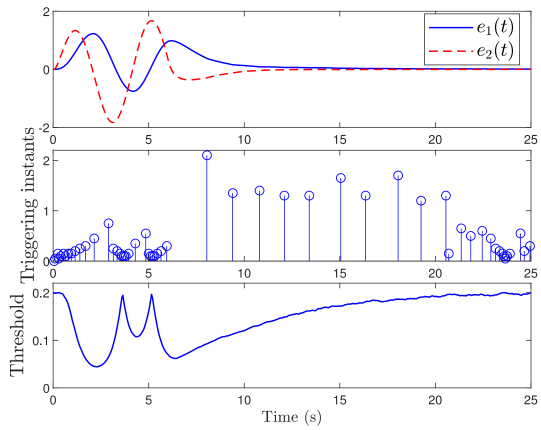

Fig. 3. Filtering error $e(t)$, triggering instants and triggering threshold parameter with $\tau = 0.5$.

TABLE II
DATA TRANSMISSION UNDER DIFFERENT SCHEMES

Method	$\mathscr{N}_1$	$\mathscr{N}_2$
Time-triggered sampling	500	100%
Conventional ETS (6)	154	30.8%
METS (4) with $\tau = 0.25$	70	14%
METS (4) with $\tau = 0.5$	55	11%

denotes the numbers of transmitted signals and $\mathscr{N}_2$ means the ratio between the number of transmitted signals under different ETSs and the number in the time-triggered sampling.

According to this table, it is obvious that compared with the conventional ETS and time-triggered scheme, the data communication quantity can be decreased dramatically through the presented METS (4). To be specific, the occupation of network bandwidth utilizing the METS with $\tau = 0.5$ is reduced by 89% and 19.8% compared with the time-triggered scheme and conventional ETS (6), respectively. Moreover, it is shown that a larger τ could lead to lower bandwidth occupation and save more network resources.

V. CONCLUSION

This brief focuses on the H_∞ filtering issue of USVs subject to communication delays under METS. To mitigate the communication burden, a new METS using the mean of past system outputs and a dynamic triggering threshold parameter has been presented. Then a united distributed delay system was established for modeling a USV system with communication delays and the METS. By applying a new integral inequality, sufficient LMI conditions were derived to design an event-triggered H_∞ filter. Finally, simulations were executed to show the advantages of the presented approach. Note that the practical system components are vulnerable to be with faults [16], which could result in system performance degradation, even system instability. Thus, the fault detection and fault-tolerant control issues based on the proposed METS deserve further investigations in the future.

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